Nonlinear Phenomena – An OV

→ So far:

- free energy (i.e. V_d)
- instability (i.e. CDIA)
- QL modification of $\langle f \rangle$ to return to marginality (i.e. as in QL description ⇒ transport)

→ But

a.) QL severely limited

- all $\omega = \omega(k)$ ⇒ eigenmodes, only.
- all $\delta f_k \cong E_k \, \partial \langle f \rangle / \partial v$ ⇒ coherent response, only
- What of nonlinear interactions?

  → harmonic generation
  → wave coupling
  ⋮

⇒ can act to couple growth/unstable modes to dissipation (damped modes)

⇒ so:

$$\partial_t |E_{\underline{k}}|^2 = 2\gamma_{\underline{k}} |E_{\underline{k}}|^2$$

becomes

$$\partial_t |E_{\underline{k}}|^2 = \overset{\text{growth}}{\overbrace{2\gamma_{\underline{k}} |E_{\underline{k}}|^2}} + \overset{\text{nonlinear transfer}}{\overbrace{\sum_{\underline{k}',\omega'} C(\underline{k},\underline{k}') |E_{\substack{\underline{k}-\underline{k}' \\ \omega-\omega'}}|^2 |E_{\substack{\underline{k}' \\ \omega'}}|^2}}$$

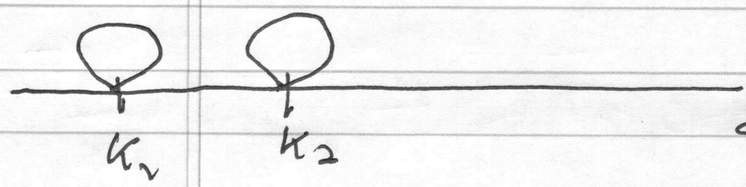
$$- \underbrace{\gamma_{\underline{k}} k^2 |E_{\underline{k}}|^2}_{\text{damping}}$$

and can, in principle, arrange stationary state with $\gamma_{\underline{k}} > 0$ (i.e. driven system).

⇒ Nonlinear transfer:

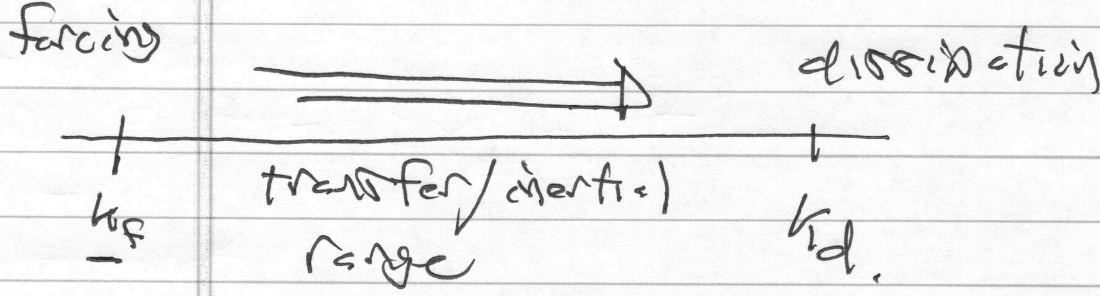
A) Contrast:

- equilibrium (ala' TPM)



emission and absorption balance at each k.

– Navier–Stokes Turbulence

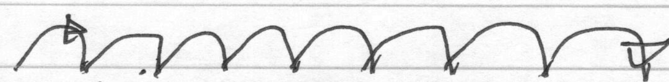


emission/input vs transfer
$\Rightarrow$ defines inertial range

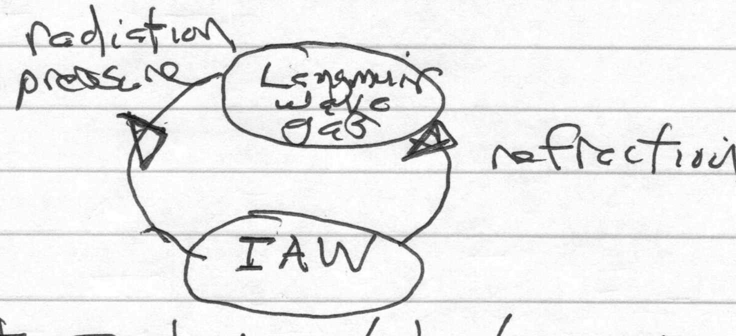
but

transfer vs. dissipation $\Rightarrow$ defines k_d.

B.) transfer can be:

– local: 

a la' NST cascade

– non-local: 

disparate scale: a la' Langmuir turbulence.

- forward or inverse:

$\longrightarrow$ (NST)

$k\uparrow$

$\longleftarrow$ (2D, Langmuir)

$k\downarrow$

- mediated by waves, particles:

i.e. $\underline{k} + \underline{k}' = \underline{k}''$

$$\omega(\underline{k}) + \omega(\underline{k}') = \omega_{\underline{k}''}$$

$\Rightarrow$ 3 wave resonance aka Fermi Golden Rule (derive via TDPT)

or

$$\omega(\underline{k}) - \omega(\underline{k}') = (\underline{k}-\underline{k}')\cdot\underline{V}$$

$\Rightarrow$ Landau resonance with beat wave

$\Rightarrow$ nonlinear Landau damping, beat wave resonance.

n.b. recall:

$$\overleftarrow{T}_{a,b}(\omega) \cong \sum_b |\langle b|e\underline{x}|a\rangle|^2 \delta(E_b - E_a - \hbar\omega)$$

$$(\hbar\omega_b - \hbar\omega_a - \hbar\omega).$$

- adiabatic approximation is powerful tool, when relevant.

Here: 2 prototypical examples:

A.)
→ Navier-Stokes Turbulence / Kolmogorov Cascade

B.)
→ Langmuir Turbulence.